1 F is inversely proportional to the square of v.

Given that $F = 6.5$ when $v = 4$

find a formula for F in terms of v.

$F \propto \frac{1}{v^2}$

$F = \frac{k}{v^2}$ (1)

when $F = 6.5$ and $v = 4$,

$6.5 = \frac{k}{4^2}$

$k = 6.5 \times 16$

$= 104$ (1)

$\therefore F = \frac{104}{v^2}$ (1)

$F = \frac{104}{v^2}$

(Total for Question 1 is 3 marks)

2 A particle moves along a straight line.
The fixed point O lies on this line.
The displacement of the particle from O at time t seconds, $t \geqslant 0$, is s metres where

$$s = t^3 + 4t^2 - 5t + 7$$

At time T seconds the velocity of P is V m/s where $V \geqslant -5$

Find an expression for T in terms of V.

Give your expression in the form $\frac{-4 + \sqrt{k + mV}}{3}$ where k and m are integers to be found.

$$v = \frac{ds}{dt}$$

$$v = 3t^2 + 8t - 5 \quad (1)$$

$$V = 3T^2 + 8T - 5$$

$$0 = 3T^2 + 8T - 5 - V \quad (1)$$

$$= \frac{-8 \pm \sqrt{(8)^2 - 4(3)(-5-V)}}{2(3)} \quad (1) \qquad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-8 \pm \sqrt{64 + 60 + 12V}}{6}$$

$$= \frac{-8 \pm \sqrt{124 + 12V}}{6} \quad (1)$$

$$= \frac{-8 \pm \sqrt{4(31 + 3V)}}{6}$$

$$= \frac{-8 \pm 2\sqrt{(31 + 3V)}}{6} \quad (1)$$

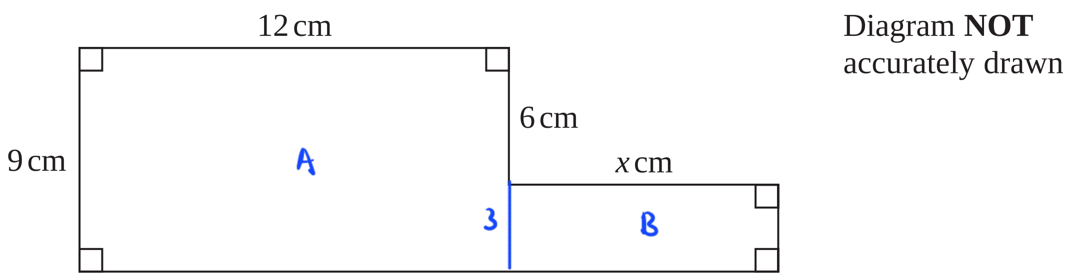
$$= \frac{-4 \pm \sqrt{31 + 3V}}{3}$$

$$\therefore \frac{-4 + \sqrt{31 + 3V}}{3}$$

$T = \frac{-4 + \sqrt{31 + 3V}}{3}$ (1)

(Total for Question 2 is 6 marks)

3 The diagram shows a shape.



The shape has area 129 cm^2

Work out the value of x.

Total Area : Area of shape A + Area of shape B

$129 = (12 \times 9) + 3x$ 1

$129 = 108 + 3x$ 1

$3x = 129 - 108$

$3x = 21$

$x = \frac{21}{3}$ 1

$= 7$ 1

$x =$ 7

(Total for Question 3 is 4 marks)

4

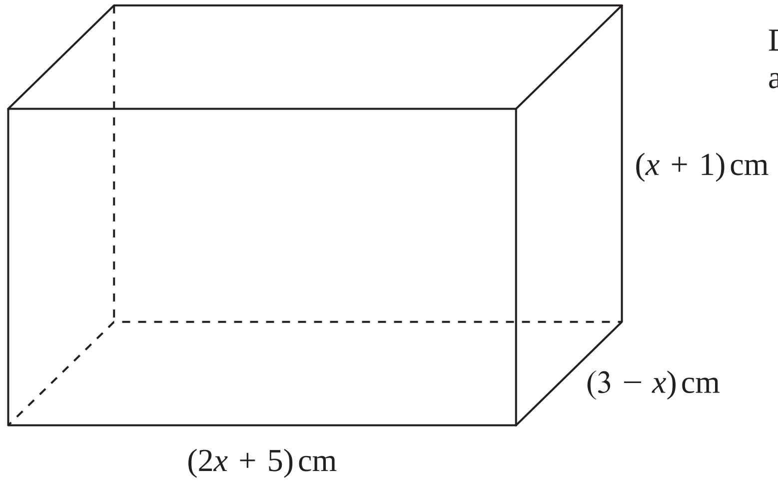


Diagram **NOT** accurately drawn

The diagram shows a cuboid of volume $V\,\text{cm}^3$

(a) Show that $V = 15 + 16x - x^2 - 2x^3$

V = length x width x height

$= (2x+5)(3-x)(x+1)$

$= (6x - 2x^2 + 15 - 5x)(x+1)$

$= (-2x^2 + x + 15)(x+1)$ 1

$= -2x^3 - 2x^2 + x^2 + x + 15x + 15$ 1

$= -2x^3 - x^2 + 16x + 15$

1

$V = 15 + 16x - x^2 - 2x^3$ (shown)

(Total for Question 4 is 3 marks)

5 A is inversely proportional to the square of r

$A = 5$ when $r = 0.3$

(a) Find a formula for A in terms of r

$A \propto \frac{1}{r^2}$

$A = \frac{k}{r^2}$ 1

when $A = 5$ and $r = 0.3$

$5 = \frac{k}{0.3^2}$

$k = 5 \times 0.3^2$

$= 0.45$ 1

$A = \frac{0.45}{r^2}$ 1

$A = \frac{0.45}{r^2}$

(3)

(Total for Question 5 is 3 marks)

6 An arithmetic series has first term a and common difference d.

The sum of the first $2n$ terms of the series is four times the sum of the first n terms of the series.

Find an expression for a in terms of d.
Show your working clearly.

$$S_{2n} = \frac{2n}{2}\left[2a + (2n-1)d\right]$$

$$= n(2a + 2nd - d) \quad (1)$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}(2a + nd - d)$$

$$S_{2n} = 4S_n$$

$$n(2a + 2nd - d) = 4\left[\frac{n}{2}(2a + nd - d)\right] \quad (1)$$

$$n(2a + 2nd - d) = 2n(2a + nd - d)$$

$$2a + 2nd - d = 2(2a + nd - d)$$

$$2a + 2nd - d = 4a + 2nd - 2d$$

$$4a - 2a = 2nd - 2nd - d + 2d \quad (1)$$

$$2a = d$$

$$a = \frac{d}{2} \quad (1)$$

a = $\frac{d}{2}$

(Total for Question 6 is 4 marks)

7 The sum of the first 10 terms of an arithmetic series is 4 times the sum of the first 5 terms of the same series.

The 8th term of this series is 45

Find the first term of this series.
Show clear algebraic working.

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{10} = \frac{10}{2}[2a+(10-1)d]$

$= 10a + 45d$

$S_5 = \frac{5}{2}[2a+(5-1)d]$

$= 5a + 10d$

$S_{10} = 4 \times S_5$

$10a + 45d = 4(5a + 10d)$ (1)

$10a + 45d = 20a + 40d$

$10a = 5d$

$d = 2a$ — (1) (1)

$T_n = a + (n-1)d$

$T_8 = 45 = a + (8-1)d$

$45 = a + 7d$ — (2) (1)

Substitute (1) into (2):

$45 = a + 7(2a)$ (1)

$45 = 15a$

$a = 3$ (1)

$\therefore$ First term, $a = 3$.

3

(Total for Question 7 is 5 marks)

8 The diagram shows a frustum of a cone, and a sphere.

The frustum, shown shaded in the diagram, is made by removing the small cone from the large cone.
The small cone and the large cone are similar.

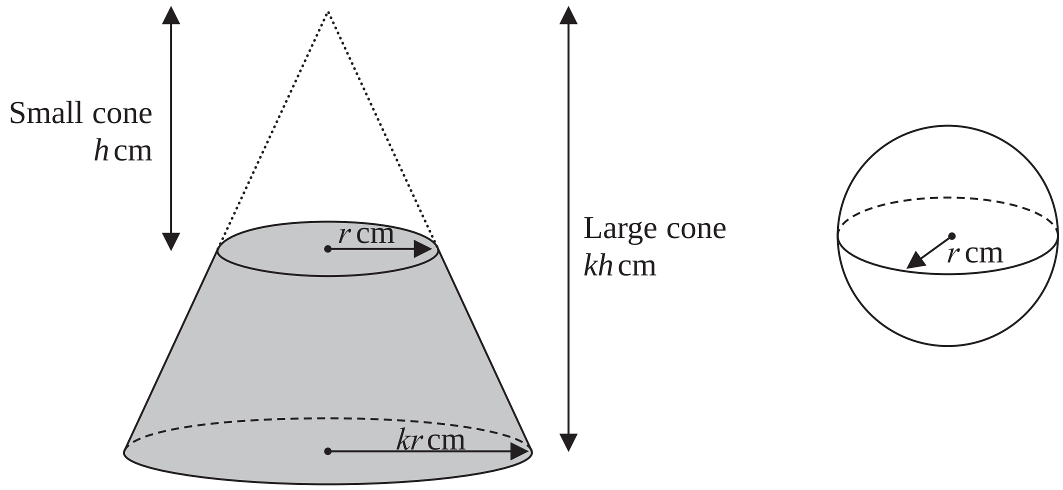


Diagram **NOT** accurately drawn

The height of the small cone is h cm and the radius of the base of the small cone is r cm.
The height of the large cone is kh cm and the radius of the base of the large cone is kr cm.
The radius of the sphere is r cm.

The sphere is divided into two hemispheres, each of radius r cm.

Solid **A** is formed by joining one of the hemispheres to the frustum.
The plane face of the hemisphere coincides with the upper plane face of the frustum, as shown in the diagram below.

Solid **B** is formed by joining the other hemisphere to the small cone that was removed from the large cone.
The plane face of the hemisphere coincides with the plane face of the base of the small cone, as shown in the diagram below.

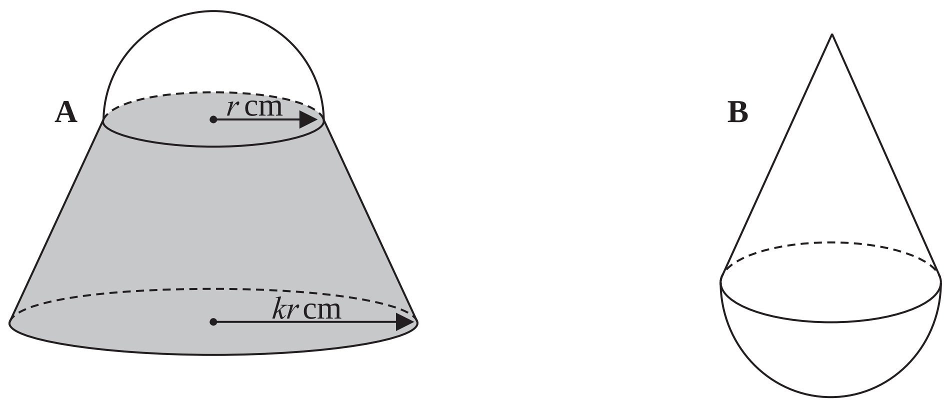


Diagram **NOT** accurately drawn

The volume of solid **A** is 6 times the volume of solid **B**.

Given that $k > \sqrt[3]{7}$

find an expression for h in terms of k and r

Volume of each hemisphere :

$\frac{1}{2} \times$ volume of sphere $= \frac{1}{2} \times \frac{4}{3} \times \pi \times r^3$

$= \frac{2}{3}\pi r^3$ (1)

Volume of small cone :

$= \frac{1}{3}\pi r^2 h$

Volume of frustrum :

Volume of large cone − volume of small cone :

$\frac{1}{3} \times \pi \times (kr)^2 \times kh - \frac{1}{3} \times \pi \times r^2 \times h$

$= \frac{1}{3}\pi r^2 h(k^3 - 1)$ (1)

Volume of Solid A :

$= \frac{1}{3}\pi r^2 h(k^3-1) + \frac{2}{3}\pi r^3$

Volume of Solid B :

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$ (1)

$\therefore \frac{1}{3}\pi r^2 h(k^3-1) + \frac{2}{3}\pi r^3 = 6\left(\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right)$ (1)

$\frac{1}{3}\pi r^2 h(k^3-1) + \frac{2}{3}\pi r^3 = 2\pi r^2 h + 4\pi r^3$

$\div \pi r^2$

$\frac{1}{3}h(k^3-1) + \frac{2}{3}r = 2h + 4r$

$h(k^3-1) + 2r = 6h + 12r$

$h(k^3-1) - 6h = 10r$ (1)

$hk^3 - 7h = 10r$

$h = \frac{10r}{k^3-7}$ (1)

$h = \frac{10r}{k^3-7}$

(Total for Question 8 is 6 marks)

9 A particle P moves along a straight line.
The fixed point O lies on this line.

The displacement of P from O at time t seconds, $t \geqslant 1$, is s metres where

$$s = 4t^2 + \frac{125}{t}$$

The velocity of P at time t seconds, $t \geqslant 1$, is v m/s

Work out the distance of P from O at the instant when $v = 0$

$v = \frac{ds}{dt} = 8t - 125t^{-2}$ (1)

when $v = 0$, $8t - \frac{125}{t^2} = 0$ (1)

$8t^3 = 125$

$t = \sqrt[3]{\frac{125}{8}}$

$= \frac{5}{2} = 2.5$ (1)

$s = 4(2.5)^2 + \frac{125}{2.5}$ (1)

$= 75$ (1)

75 .... m

(Total for Question 9 is 5 marks)

10

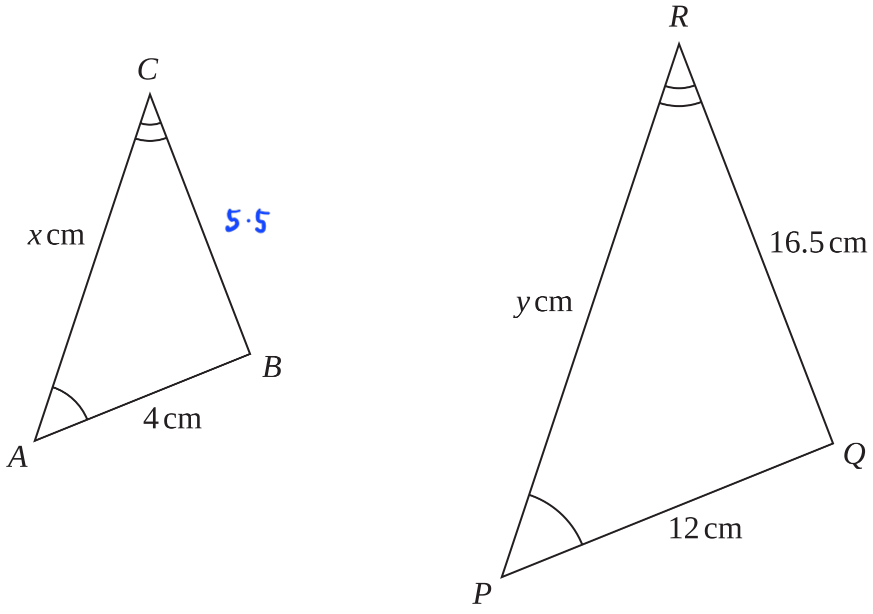


Diagram **NOT** accurately drawn

(b) Write down an expression for y in terms of x

$y = 3x$

$y =$ ....$3x$.......... (1)

(1)

(Total for Question 10 is 1 marks)

11 The diagram shows a solid cone and a solid sphere.

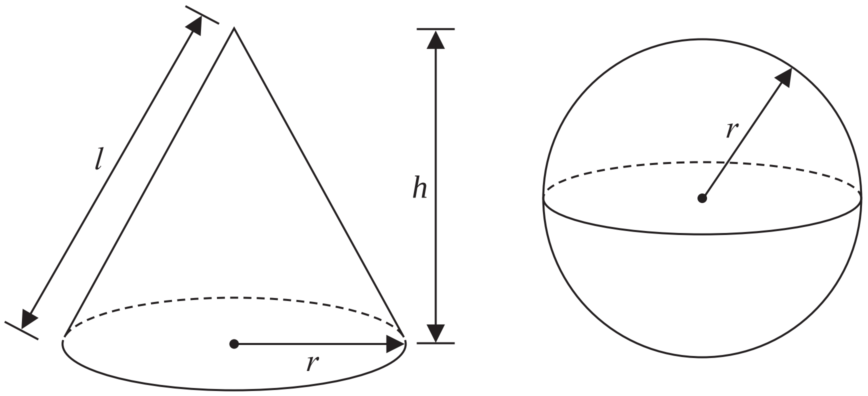


Diagram **NOT** accurately drawn

The cone has base radius r, slant height l and perpendicular height h
The sphere has radius r

The base radius of the cone is equal to the radius of the sphere.

Given that

$$k \times \text{volume of the cone} = \text{volume of the sphere}$$

show that the **total** surface area of the cone can be written in the form

$$\pi r^2 \left(\frac{k + \sqrt{k^2 + a}}{k} \right)$$

where a is a constant to be found.

$k \times \frac{1}{\cancel{3}} \times \cancel{\pi} \times \cancel{r^2} \times h = \frac{4}{\cancel{3}} \cancel{\pi} \times \cancel{r^3}$ 1

$kh = 4r$

$h = \frac{4r}{k}$ 1

$l^2 = h^2 + r^2$

$= \left(\frac{4r}{k}\right)^2 + r^2$ 1

$l = \sqrt{r^2 + \left(\frac{4r}{k}\right)^2}$

$$l = \sqrt{r^2\left(1 + \frac{16}{k^2}\right)}$$

$$l = r\sqrt{\frac{k^2 + 16}{k^2}}$$

$$l = r\frac{\sqrt{k^2+16}}{k} \quad (1)$$

Total surface area : $\pi r^2 + \pi r l$

$$= \pi r^2\left(1 + \frac{\sqrt{k^2+16}}{k}\right) \quad (1)$$

$$= \pi r^2\left(\frac{k + \sqrt{k^2+16}}{k}\right) \quad (1)$$

(Total for Question 11 is 6 marks)

12 Given that

$$2^n = 2^{x^2} \times 16^x \times 8$$

and

$$x > 0$$

find an expression for x in terms of n
State any restrictions on n

$16 = 2^4$ (1)

$8 = 2^3$

$2^n = 2^{x^2} \times 2^{4x} \times 2^3$

$n = x^2 + 4x + 3$ (1)

$n = (x+2)^2 - 4 + 3$

$n = (x+2)^2 - 1$ (1)

$n+1 = (x+2)^2$

$\pm\sqrt{n+1} = x+2$

$x = -2 \pm \sqrt{n+1}$ (1)

since $x > 0$, $-2+\sqrt{n+1}$ only

let $x = 0$, $0 = -2 + \sqrt{n+1}$

$2 = \sqrt{n+1}$

$4 = n+1$

$n = 3$

$\therefore n > 3$ for $x > 0$

$x = -2 + \sqrt{n+1}$ and $n > 3$. (1)

(Total for Question 12 is 5 marks)